# Setting the scale for DIS at large Bjorken x

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**Abstract.** We discuss the extension of a systematic perturbative QCD based analysis to the $x \to 1$ region. After subtracting a number of effects that transcend NLO pQCD evolution, such as target mass corrections and large $x$ resummation effects, the remaining power corrections can be interpreted as dynamical higher twists. The quantitative outcome of the analysis is dominated by the interplay between the value of $\alpha_S$ in the infrared region and the higher twists. We uncover a dual role played by $\alpha_S$ at large Bjorken $x$ that can be used to experimentally extract its value in the non-perturbative regime.

**Keywords:** nucleon structure functions, momentum transfer dependence, quantum chromodynamics: perturbation theory, numerical calculations: interpretation of experiments
**PACS:** 25.30.Dh, 25.30.F,12.38.Bx, 12.38.Cyh

## INTRODUCTION

*"QCD nowadays has a split personality. It embodies 'hard' and 'soft' physics, both being hard subjects and the softer the harder."* [1]

QCD's main goal is to describe the structure of hadrons in terms of its fundamental degrees of freedom, the quarks and gluons (partons). Hadrons are observable in both the initial and final stages of hard processes, while the existence and properties of partons are inferred only indirectly. The underlying idea is that because of the smallness of the coupling constant at large enough momentum transfer, $Q^2$, or equivalently at short distances $\mathcal{O}(1/\sqrt{Q^2})$, a hard probe sees hadrons as composed of "free" quarks and gluons carrying fractions $x$ of the hadron's Light Cone (LC) momentum, with given probability distributions. In Deep Inelastic Scattering (DIS) the latter are identified with the Parton Distribution Functions (PDFs) [2]. As increasingly shorter distances are probed the PDFs shape in $x$ changes due to radiative processes, according to a pattern which is calculable with high accuracy within Perturbative QCD (PQCD) [3]. Factorization theorems regulate this fundamental property of the theory thereby allowing for the short distance behavior to be evaluated using as input the values taken by the PDFs at a given initial $Q^2$. The strong coupling regime of QCD remains incalculable; it is accessible only with non-perturbative (approximate) methods.

The separation and yet coexistence of long distance and short distance structure in QCD has by now become naturally accepted as part of a "common wisdom framework" underlying the interpretation of most experiments, from Deep Inelastic Scattering (DIS) to $e^+e^- \to$ hadrons, to hadron-hadron scattering. The concept of *duality* is implicitly used to various degrees, meaning, in the most extreme case, that hadronic observables are replaced by calculable partonic ones with little more going into the hadronic formation phase of the process (from partons to hadrons or vice versa). In a phenomenological

context, duality purports to study how a number of properties defined from the beginning of the hard scattering process, are predetermined and persist in the non-perturbative stage. A particularly striking realization of duality, known as Bloom Gilman duality [4], is observed in DIS, where for large values of Bjorken $x > 0.5$ ($x = Q^2/2M\nu$, $M$ being the proton mass and $\nu$ the energy transfer in the lab system), and for $Q^2 \approx 5$ GeV$^2$, one has an invariant mass of $W^2 \leq 5$ GeV$^2$ ($W^2 = Q^2(1/x - 1) + M^2$), lying mostly in the resonance region. While it is impossible to reconstruct the detailed structure of the proton's resonances, these remarkably follow the PQCD predictions when averaged over $x$ (see *e.g.* [5] for a review).

Duality in inclusive scattering is clearly a phenomenological manifestation of the non-perturbative to perturbative transition in QCD, whose origins are still largely unknown. It is now becoming mandatory to have a fuller understanding of its working, motivated on one side by the existence of highly accurate data at large $x$, and spurred, on the other, by the advent of the LHC where previously unexplored regimes in $x$ and $Q^2$ will be within reach.

Our studies of the physical origin of duality, and of its impact on our understanding of the nucleon's structure started a few years ago when we set up a program to quantitatively extract the scale dependence of the large $x$ Jlab Hall C data [6]. In the analysis performed in Refs.[7, 8] we addressed several effects that have a large impact at large $x$, namely Target Mass Corrections (TMCs), large $x$ resummation effects, and higher twists. This work was then completed, and extended to polarized data in Ref.[9]. An important point emerged from Refs.[7, 8, 9] that a deeper understanding was needed of those aspects unique to the large $x$ perturbative QCD analysis. Our point of view is that only after a complete perturbative QCD analysis is performed can one *define* duality by quantitatively establishing whether, and to what extent, this phenomenon is responsible for the apparent cancellation of multiparton correlations. This point of view is somewhat complementary to the approaches of Refs.[10, 11, 12], which are quark model based and focused on symmetry aspects.

In this contribution we argue that once the range of validity of parton-hadron duality is defined quantitatively, a precise PQCD analysis at large $x$ would open up the possibility of extracting the strong coupling constant, $\alpha_S$, at low scale. Such an analysis would complement the recent extractions using data on the GDH sum rule [13]. It would, furthermore, add insight on a recent interpretation put forth in Ref.[14] of the effective coupling constant in the strongly interacting/non perturbative regime from light front holographic mapping of classical gravity in Anti de-Sittter (AdS) space.

In order to explain our approach, we first present an overview of the large $x$ data, and discuss a few aspects of the evolution mechanism for DIS at large $x$ where two scales related to the invariant mass and to the four-momentum transfer, are simultaneously present. We reiterate that accurate analyses in this region such as the ones first conducted *e.g.* in [7, 8, 9] are crucial for establishing the interplay of the various components ($\alpha_S$, multiparton correlations, etc..) of the perturbative to non-perturbative transition regime in QCD. We then illustrate the connection between large $x$ data and $\alpha_S$ in the infrared region, and draw our conclusions.

## ANALYSIS OF LARGE $x$ DATA

High precision inclusive unpolarized electron-nucleon scattering data on both hydrogen and deuterium targets from Jefferson Lab are available to date in the large $x$, multi-GeV regime (see [15] and references therein). Because of the precision of the data one should now be able to distinguish among different sources of scaling violations affecting the structure functions in addition to standard NLO evolution,

- Target Mass Corrections (TMC),
- Large $x$ Resummation Effects (LxR)
- Nuclear Effects
- Dynamical Higher Twists (HTs),
- Impact of Next-to-Next-to-Leading-Order (NNLO) perturbative evolution.

All of the effects above can be extracted with an associated theoretical error. It is in fact well known that their evaluation is model dependent. Recent studies, however, have been directed at determining more precisely both the origin and size of the associated theoretical error. Recent analyses have been taking into account, so far, some but not all of the effects listed above [16].

### Unpolarized structure function.

The inclusive DIS cross section of unpolarized electrons off an unpolarized proton is written in terms of the two structure functions $F_2$ and $F_1$,

$$\frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2}{Q^2 xy}\left[\left(1-y-\frac{(Mxy)^2}{Q^2}\right)F_2 + y^2 x F_1\right], \tag{1}$$

with $y = \nu/\varepsilon_1$, $\varepsilon_1$ being the initial electron energy. The structure functions are related by the equation

$$F_1 = F_2(1+\gamma^2)/(2x(1+R)), \tag{2}$$

where $\gamma^2 = 4M^2x^2/Q^2$, and $R$ is ratio of the longitudinal to transverse virtual photo-absorption cross sections. In QCD, $F_2$ is expanded in series of inverse powers of $Q^2$, obtained by ordering the matrix elements in the DIS process by increasing twist $\tau$, which is equal to their dimension minus spin

$$F_2(x,Q^2) = F_2^{LT}(x,Q^2) + \frac{H(x)}{Q^2} + \mathscr{O}\left(\frac{1}{Q^4}\right) \simeq F_2^{LT}(x,Q^2)\left(1+\frac{C(x)}{Q^2}\right) + \mathscr{O}\left(\frac{1}{Q^4}\right) \tag{3}$$

The first term is the leading twist (LT), with $\tau = 2$. The terms of order $1/Q^{\tau-2}$, $\tau \geq 4$, in Eq.(3) are the higher order terms, generally referred to as higher twists [2].

*Target Mass Corrections*

TMCs are included in $F_2^{LT}$. For $Q^2 \geq 1$ GeV$^2$, TMCs can be taken into account through the following expansion [17]

$$F_2^{LT(TMC)}(x,Q^2) = \frac{x^2}{\xi^2\gamma^3} F_2^\infty(\xi,Q^2) + 6\frac{x^3M^2}{Q^2\gamma^4}\int_\xi^1 \frac{d\xi'}{\xi'^2} F_2^\infty(\xi',Q^2), \tag{4}$$

where $F_2^\infty$ is the structure function in the absence of TMCs. A more recent analysis [18] re-examined TMCs within the collinear factorization approach of [19] in order to address the longstanding question of the unphysical behavior in the threshold region of Eq.(4). This originates from the fact that as $x \to 1$, one obtains a $Q^2$-dependent threshold, namely $F_2(\xi,Q^2) = 0$ for $\xi > \xi_{max} = 2/1+\sqrt{1+4M^2/Q^2}$, therefore rendering $F_2$ undefinable as $Q^2$ varies (see discussion in [20]). In the formalism of Ref.[18], TMCs, applied to the helicity dependent structure functions read

$$F_T^{LT(TMC)}(x,Q^2) = \int_{m_\pi^2}^{\frac{1-x}{x}Q^2} dm_J^2 \rho(m_J^2) F_T^\infty\left[\xi\left(1+\frac{m_J^2}{Q^2}\right),Q^2\right], \tag{5}$$

where $F_T \equiv F_1$. The final quark is assumed to hadronize into a jet of mass $m_J$ with a a process dependent distribution/smearing function $\rho(m_J^2)$. In our extraction we take the perspective that the evaluation of TMCs is always associated with the evaluation of HTs – TMCs should in principle be applied also to HTs – in an inseparable way. Therefore we consistently keep terms of $\mathscr{O}(1/Q^4)$ [9, 21], whether in the formalism/prescription of Ref.[17] or of Ref.[18]. $H(x,Q^2)$, then, represents the "genuine" HT correction that involves interactions between the struck parton and the spectators or, formally, multi-parton correlation functions.

*Threshold Resummation*

In order to understand the nature of the remaining $Q^2$ dependence that cannot be described by NLO pQCD evolution, we also include the effect of threshold resummation, or Large $x$ Resummation (LxR). LxR effects arise formally from terms containing powers of $\ln(1-z)$, $z$ being the longitudinal variable in the evolution equations, that are present in the Wilson coefficient functions $C(z)$. Below we write schematically how the latter relate the parton distributions to *e.g.* the structure function $F_2$,

$$F_2^{LT}(x,Q^2) = \frac{\alpha_s}{2\pi}\sum_q \int_x^1 dz C(z)\, q(x/z,Q^2), \tag{6}$$

where we have considered only the non-singlet (NS) contribution to $F_2$ since only valence quarks distributions are relevant in our kinematics. The logarithmic terms in $C(z)$ become very large at large $x$, and they need to be resummed to all orders in $\alpha_S$. Resummation was first introduced by linking this issue to the definition of the correct kinematical variable that determines the phase space for the radiation of gluons at large $x$. This was found to be $\widetilde{W}^2 = Q^2(1-z)/z$, instead of $Q^2$ [22, 23]. As a

result, the argument of the strong coupling constant becomes $z$-dependent: $\alpha_S(Q^2) \rightarrow \alpha_S(Q^2(1-z)/z)$ [24, 25]. In this procedure, however, an ambiguity is introduced, related to the need of continuing the value of $\alpha_S$ for low values of its argument, *i.e.* for $z \rightarrow 1$ [26]. Although on one side, the size of this ambiguity could be of the same order of the HT corrections and, therefore, a source of theoretical error, on the other by performing an accurate analysis such as the one proposed here, one can extract $\alpha_S$ for values of the scale in the infrared region. We address this point in more detail in the next Section.

### *Nuclear Effects*

Theoretical uncertainties in the deuteron are taken routinely into account, and are expected to be in sufficient control (see [27] and references therein). Uncertainties arise mainly from

*i)* Different models of the so called nuclear EMC effect;

*ii)* Different deuteron wave functions derived from currently available NN potentials, giving rise to different amounts of high momentum components;

*iii)* The interplay between nucleon off-shellness and TMC in nuclei.

Finally, we did not consider NNLO calculations, these are not expected to alter substantially our extraction since, differently from what seen originally in the case of $F_3$, these have been proven to give a relatively small contribution to $F_2$.

Once all of the above effects have been subtracted from the data, and assuming the validity of the twist expansion, Eq.(3) in this region, one can interpret more reliably any remaining discrepancy in terms of HTs. Since we extend our $x$-dependent analysis to the resonance region we consider the following integrated quantities

$$I^{\rm res}(\langle x \rangle, Q^2) = \int_{x_{\rm min}}^{x_{\rm max}} F_2^{\rm res}(x, Q^2)\, dx \qquad (7)$$

where $F_2^{\rm res}$ is evaluated using the experimental data in the resonance region. For each $Q^2$ value: $x_{\rm min} = Q^2/(Q^2 + W^2_{\rm max} - M^2)$, and $x_{\rm max} = Q^2/(Q^2 + W^2_{\rm min} - M^2)$, where $W_{\rm min}$ and $W_{\rm max}$ delimit the resonance region, and $\langle x \rangle$ is the average value of $x$ for each kinematics. This procedure replaces a strict point by point in $x$, analysis.

Typical results from the analysis outlined above are plotted in Figure 1 where we show the HT coefficient defined from Eq.(3) as

$$R^{LT} \equiv C(x) = Q^2 \left[ F_2(x, Q^2)/F_2^{LT}(x, Q^2) - 1 \right]$$

The error in the figure is from the experimental data. No theoretical uncertainty was included. However, our results clearly show that the combined effects of TMCs and LxR substantially reduce $C(x)$. We take this as illustrative of the accomplishments one can expect from the analysis we suggest in this contribution. Essential features that emerge are the interplay between the values of $\alpha_S(M_Z^2)$ and the HTs, the relevance of TMCs, and, most importantly, the need to define $\alpha_S$ in the infrared region. All of these features can affect the central values of the HTs reported in Fig.1.

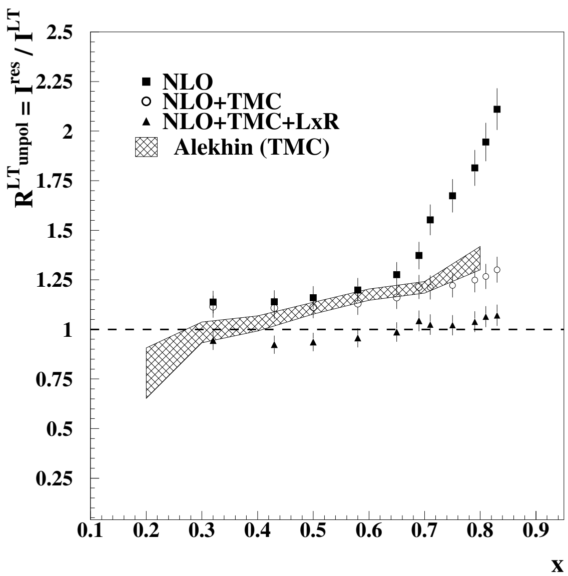


**FIGURE 1.** HT coefficients extracted in the resonance region according to the procedure described in the text. HT extracted with only the NLO calculation (squares); the effect subtracting TMC (open circles); the effect of subtracting both TMC and LxR (triangles). Shown for comparison are the values obtained from the coefficient $H$ obtained in Ref.[28] using DIS data and including the effect of TMC. Adapted from Ref.[9]

## $\alpha_S$ AT $x \to 1$

We now discuss in more detail the working of threshold resummation, and its possible impact on the analysis of $F_2$ at large $x$ [24]. Starting from NLO, the coefficient, $C(z)$ in Eq.(6) is dominated at large $x$ by terms proportional to $[\alpha_S(Q^2)\ln(1-z)]^n$ which need to be resummed in the perturbative series. The physical origin of these terms is in the phase space for the contribution of gluons emission to evolution, which become soft as $x \to 1$. A mismatch in the cancellation with the virtual gluons contributions ensues. If, however, one carefully evaluates the kinematics for gluon emission at large $x$ within a quark-parton model view, one obtains [22]

$$q(x,Q^2) \;=\; q(x,Q_o^2) + \int_{Q_o^2}^{\widetilde{W}^2} \frac{dk_\perp^2}{k_\perp^2} \frac{\alpha_S(k_\perp^2)}{2\pi} \int_x^1 \frac{dz}{z} P_{qq}\left(\frac{x}{z}, \alpha_S\right) q(z,k_\perp^2), \qquad (8)$$

where $Q_o^2$ is an arbitrary initial scale, and $\widetilde{W}^2 = Q^2(1-z)/z$ is the maximum $k_\perp^2$ in the virtual photon-quark center of mass system, appearing in the ladder graphs that define the leading log result. The resulting phase space is shown for different $Q^2$ values in Fig.2. The reduction of the allowed $k_\perp^2$ results in a simultaneous shift in the argument of $\alpha_S \to \alpha_S(Q^2/(1-z)/z)$, and a cancellation of the $\alpha_S(Q^2)\ln(1-z)$ divergence in the NLO coefficient function. As a consequence of rescaling the argument of $\alpha_S$ one has to consider its continuation into the infrared region [25, 9]. The left panels of Fig.3 display the results for $\alpha_S$ used in our analysis for different values of $Q^2$. We also show, on the right, the extracted value of the effective $\alpha_S$ from the GDH sum rule. We therefore

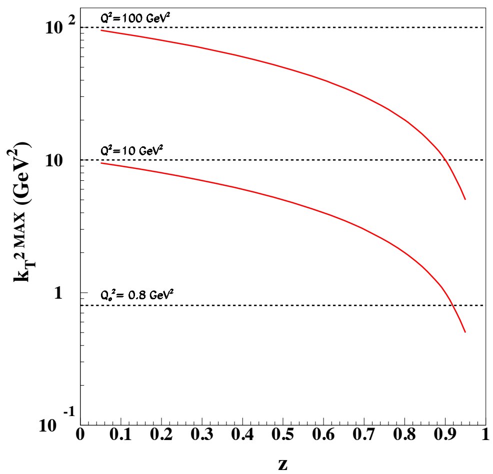


**FIGURE 2.** Phase space in the evolution of the NS component. The dotted lines are for $k^2_{T,MAX} = Q^2$, for $Q^2 = 0.8, 10, 100$ GeV$^2$. The full lines represent the upper limit $k^2_{T,MAX} = Q^2(1-z)/z$ for $Q^2 = 10, 100$ GeV$^2$. In this case, one can see a clear reduction of the allowed $k_T$ at large $z$.

suggest large $x$ evolution in DIS as yet another way of defining an effective coupling constant at low values of the scale. A more quantitative analysis to relate different types of measurements, and to study in depth the possible process dependence of $\alpha_S$ is in progress [29].

In conclusion, we believe there is a much richer structure to the scale dependence of the nucleon's distribution functions that persists behind the apparent cancellation among higher twist terms. We started uncovering this structure in the initial work of Refs.[7, 9]. While on one side this points at the fact that PQCD provides an essential framework for understanding the working of duality, on the other a thorough understanding of the lack of final state interactions is still missing. Our analysis opens up the possibility of extracting the effective strong coupling constant, $\alpha_S$, at low scale from a different process than in Refs.[13, 14].

This work is supported by the U.S. Department of Energy grant no. DE-FG02-01ER41200.

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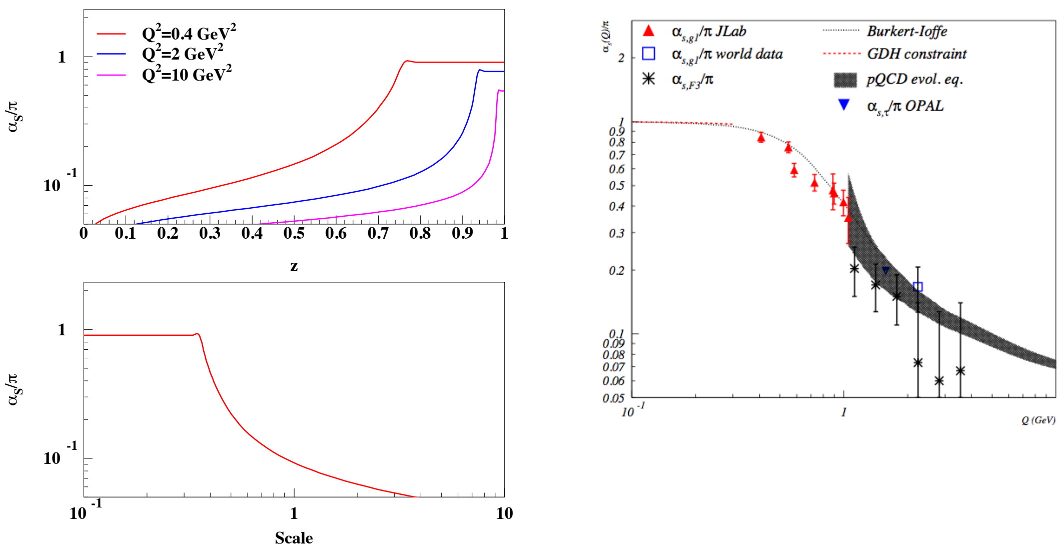


**FIGURE 3.** Left panel: $\alpha_S/\pi$ extracted from the analysis of the large $x$ data discussed in the text, and plotted vs. $z$ (Eq.(6) (upper panel), and the scale $\widetilde{W} = \sqrt{Q^2(1-z)/z}$ (lower panel). For comparison we show the extraction from Ref.[13] using Jefferson Lab data at $Q^2 = 0.7 - 1.1$ GeV$^2$.


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